B.Sc. - II (CBCS Pattern) Semester-III

USMT-05 - Mathematics-I : Real Analysis

P. Pages : 2 **GUG/S/25/11612**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Let $X=\langle x_n\rangle$ and $Y=\langle y_n\rangle$ be sequences of real numbers that converges to x and y respectively. Then prove that the sequence $X+Y$ converges to $x+y$. **6**

b) Find the limit of the sequence $\langle S_n\rangle$ where **6**

$$S_n=\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\dots\dots+\frac{1}{(n+n)^2}$$

OR

c) Let $\langle S_n\rangle$ be a sequence such that $S_n>0\,\forall n\in N$. Then prove that **6**

$$\lim_{n\to\infty}(S_n)^{1/n}=\lim_{n\to\infty}\frac{S_n}{S_{n-1}}.$$

Provided the limit on the right side exists.

d) Show that the sequence $\langle S_n\rangle$, where $S_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\dots+\frac{1}{n!}$ is convergent. **6**

UNIT – II

2. a) Show that a geometric series $\sum_{n=1}^{\infty}x^{n-1}$ converges to $\frac{1}{1-x}$ for $0<x<1$ and diverges for $x\ge 1$. **6**

b) Test the convergence of following series Σx_n by direct Comparison Test where- **6**

i) $x_n=\frac{1}{n^2+2}$ ii) $x_n=\frac{1}{5n-4}$

OR

c) Show that $\sum\frac{1}{(2n+1)^3}$ is convergent and $\sum\frac{1}{(2n-1)^{1/2}}$ is divergent. **6**

d) Test the Convergence of the series. **6**

$$\sum\frac{n^3+a}{2^n+a}$$

UNIT – III

3. a) Show that $d(x,y)=|x-y|,\forall x,y\in R$ defines a metric on R. **6**

b) Prove that every neighbourhood is an open set. **6**

OR

c) Prove that finite intersection of open sets is open. Give counter example to show that arbitrary intersection of open sets need not be open. **6**

d) Prove that every convergent sequence in a metric space is a Cauchy Sequence. **6**

UNIT – IV

4. a) If f is a bounded and integrable function over [a, b] and M, m are the bounds of f over [a, b], then prove that. **6**

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

b) If f is a function defined by $f(x) = x$ on $[0, 2]$, then show that f is integrable in Riemann sense over [0, 2] and $\int_0^2 4(x)dx = 2$. **6**

OR

c) Let f be continuous and non-negative on [a, b]. Then prove that $F(x) = \int_a^x f(t)dt$ is monotonic increasing in [a, b] further more $\int_a^b f(t)dt = F(b) \ge 0$ and equality holds true only for f identically zero on [a, b]. **6**

d) Prove that the inequality **6**

$$\frac{2}{17} < \int_1^2 \frac{x}{1+x^4} dx < \frac{1}{2}$$

5. Solve **any six.**

a) Find the n^{th} term of the sequence **2**
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

b) State Sandwich theorem on sequence. **2**

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ **2**

d) Test the convergence by P-series test $\Sigma \frac{1}{n^3}$ **2**

e) Define closed set. **2**

f) Define Cauchy sequence in metric space. **2**

g) Define Darboun's upper and lower suns. **2**

h) For any partition P prove that **2**
$L(p, f) \le U(p, f)$.

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